The Weibull Renewal Process Model

We Know Lobsters Smell Good, But Do We Know How They Smell?

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Introduction

In order to interact with the world, organisms have developed sensory organs that are capable of detecting external stimuli and transferring that signal to the brain. These signals are then relayed through neurons which relay the information through the use of electrical signals. A stimulus is capable of indirectly inducing a current onto a neuron which can cause the cell to depolarize. If the depolarization is strong enough (surpasses a given threshold) an action potentials or 'spike' is produced. It is this spike that then propagates along the neuron, sending signals to nearby cells and conveying information about the applied stimulus. Here we investigate the olfactory system, in that we are looking at how a lobster is able to detect scent from a given order via its antennule (Figure 1a) through their bursting olfactory response neurons (bORNs).

Electrophysiological monitoring and calcium imaging were used to detect the activity of bORNs (Figure 2a/2b). It has been show that in the absence of any odors the bORNs probabilistically fire, at a relatively low frequency, depending on the time since the last burst. The spontaneous firing was described as a renewal process following a truncated normal distribution. When an odor is introduced, the neuron will fire in response to the odor probabilistically depending on the time since the last burst. This evoked firing was shown to have a given probability of firing described by a logistic function and dependent on the time since the last burst. Initially a model was made that treated the system as a renewal point process that was described by the neurons spontaneous and evoked bursting probability (Figure 1b). Although a strong model, it relies on a all-or-none response to an odor and is invariant to odor concentration. We propose a **new renewal point process model that is able to deal with these low oscillatory signals with a varying stimulus and who spontaneous inter-burst interval (IBI) can be described nicely by a Weibull Distribution and whose negative likelihood is nonconvex.**

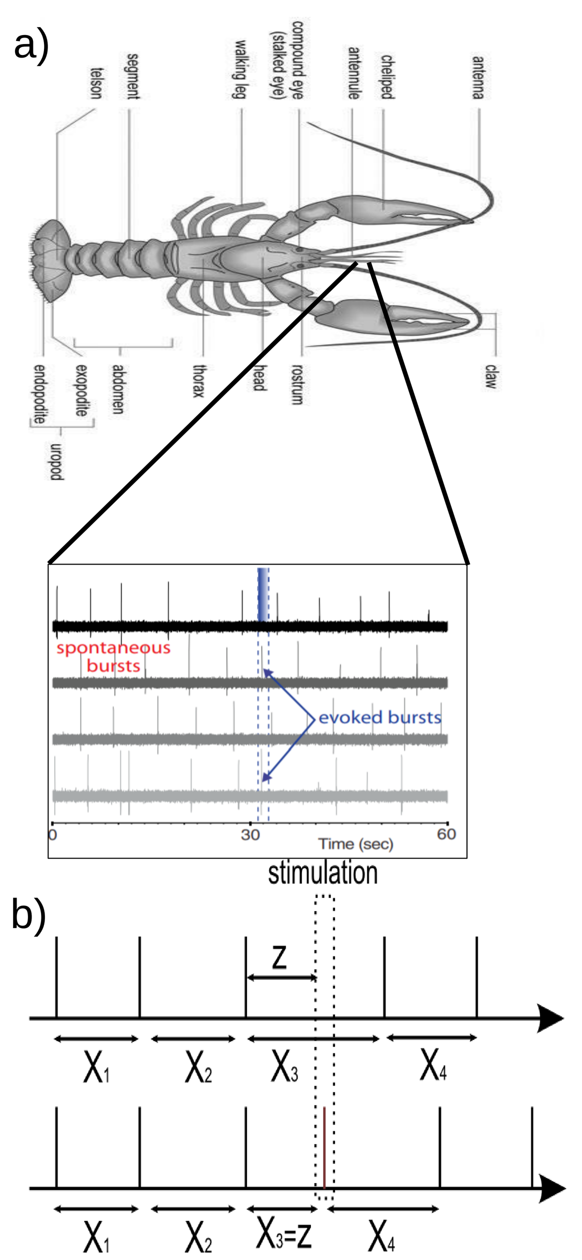

Fig 1: a) **top:** Anatomical structure of a lobster. Recordings where taken in situ of antennule. **Bottom:** Electrophysical recordings over multiple trials from the antennule in the presence of an odor. b) Overview of previous renewal point process model.

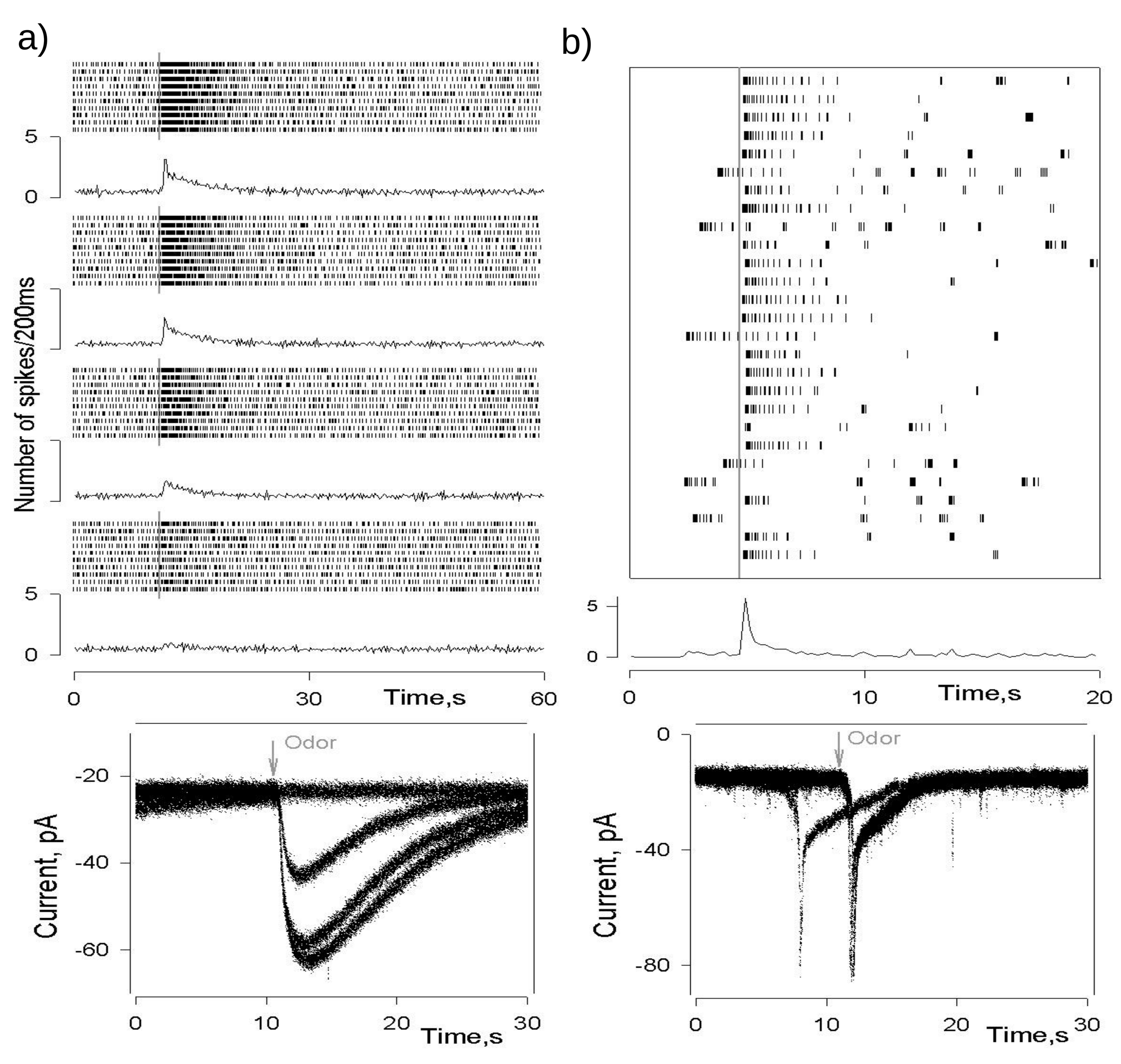

Fig 2: a) **Top:** Raster display of action potentials taken in situ of antennule over a series of trials with the same stimulus at the same time (gray line) and over a series of different odor concentrations. b) **Top:** Zoomed in raster plot around the timing of an odor (gray line). **Middle**: Corresponding Peri-stimulus time histogram displaying average response to the stimulus. **Bottom Panels:** Whole cell ORN response at a holding potential of -70 mv.

WRP Model in Relation to Fano Factor

This model can handle both under dispersed and over dispersed data depending on the parameter k as described below and shown in figure 6:

$$Fano\ Factor = \frac{\Gamma\,(1+\frac{2}{k})}{\Gamma^{\,2}(1+\frac{1}{k})} - 1$$

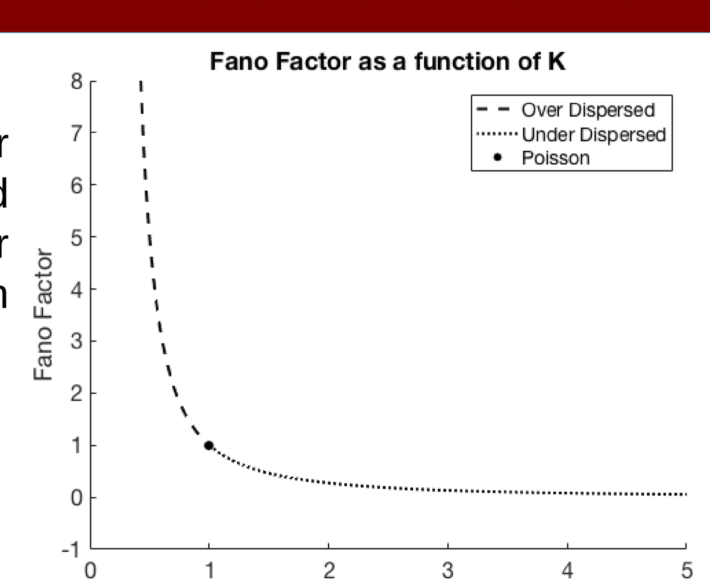

Fig 6: Relationship between the Fano Factor and the WRP model. The model indicates that the dispersion is solely dependent on the k parameter. If k is less then 1 the data is over dispersed. If k is greater then 1 the data is under dispersed. If the k is 1, the model reduces to that of a Poisson distribution.

The Weibull Renewal Process Model

The Weibull Renewal Process Model (WRP) can be described by its conditional intensity function (See figure 4 red):

$$\lambda(t|a,k,H_t) = \begin{cases} a * v(t)^{k-1} & if\ v(t) \geq 0 \\ 0 & otherwise \end{cases}$$

Where **a** and **k** are parameters, t is the time and v is described below (See figure 4 blue):

$$v(t) = t - t^* + \sum_{s=t^*}^{t} \sum_{u=0}^{bmax} x(s-u)b(u)$$

Where t^* is the time of the last spike and b is the stimulus filter.

In the spontaneous regime the Inter-burst interval (IBI) is described by the Weibull Distribution:

$$f(x) = \frac{k}{\rho^k}\left(\frac{v(t)}{\rho}\right)^{(k-1)} e^{\left(-\frac{v(t)}{\rho}\right)^k}$$

Where $\rho = (^k/_a)^{(1/_k)}$

The WRP model (summarized in figure 3 top) shares properties with that of a Generalized Linear Model (GLM) whose conditional intensity function is:

$$\lambda(t) = \exp\left(\sum_{\tau} H(\tau)y(t-\tau) + \sum_{u} x(t-u)b(u) + \beta\right)$$

Where H is a history filer and β is bias. The WRP differs from that of the Generalize Linear Model (figure 3 bottom) in that it does not rely on a history filter, but rather is reset after each spike.

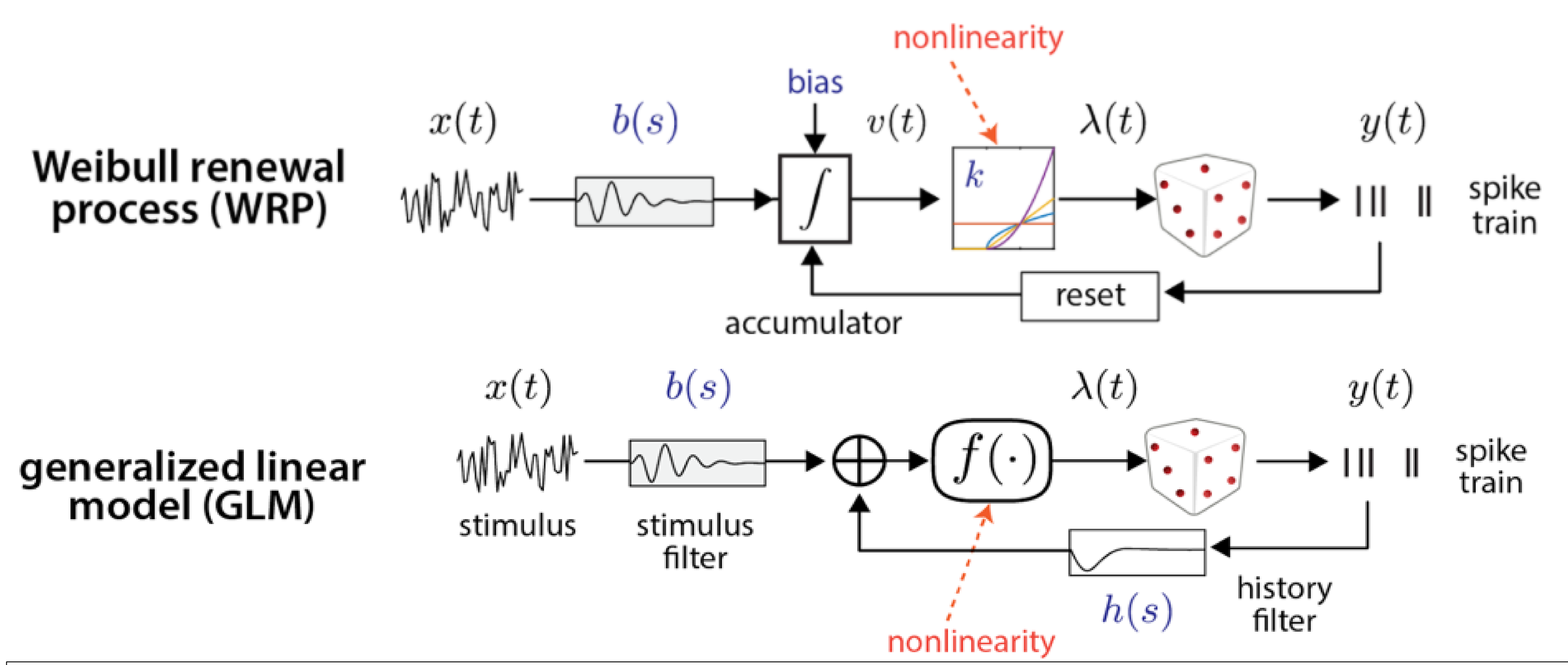

Figure 3: **Top**: Schematic of the WRP model. Stimulus is filtered through a stimulus filter. The stimulus is integrated into the 'membrane potential' then processed through a nonlinear conditional intensity function followed by some poison spiking. The system is then reset and start over again. **Bottom:** Schematic of a GLM where similar processing is done except after spikes are generated there is no reset but instead a history filter is used to deal with past spiking behavior.

Parameter Estimation was done via Maximum Likelihood Estimation. For a Point Process described by its conditional intensity function, its Likelihood can be described as:

$$LL(\theta) \approx \sum_{t} y \log(\lambda(t|H_t)\Delta) - \lambda(t|H_t)\Delta$$

Where y is a binary set of data where a 1 means a spike occurred, and delta is a small time bin.

The parameter derivatives can be described as:

$$\frac{\partial LL(\theta)}{\partial \theta} = \sum_{t} \frac{\partial \lambda(t)}{\partial \theta}\left(\frac{y(t)}{\lambda(t)} - \Delta\right)$$

Where θ is either **a, k or b.**

After Likelihood estimation, our model was then compared to a null model (homogenous Poisson model) through a pseudo r^2 as shown below:

$$r^2 = \frac{LL - LL_o}{LL_{sat} - LL_0} * 100$$

Where LL_0 is the null model and Ll_{sat} is a saturated model (described by the mean firing rate per bin).

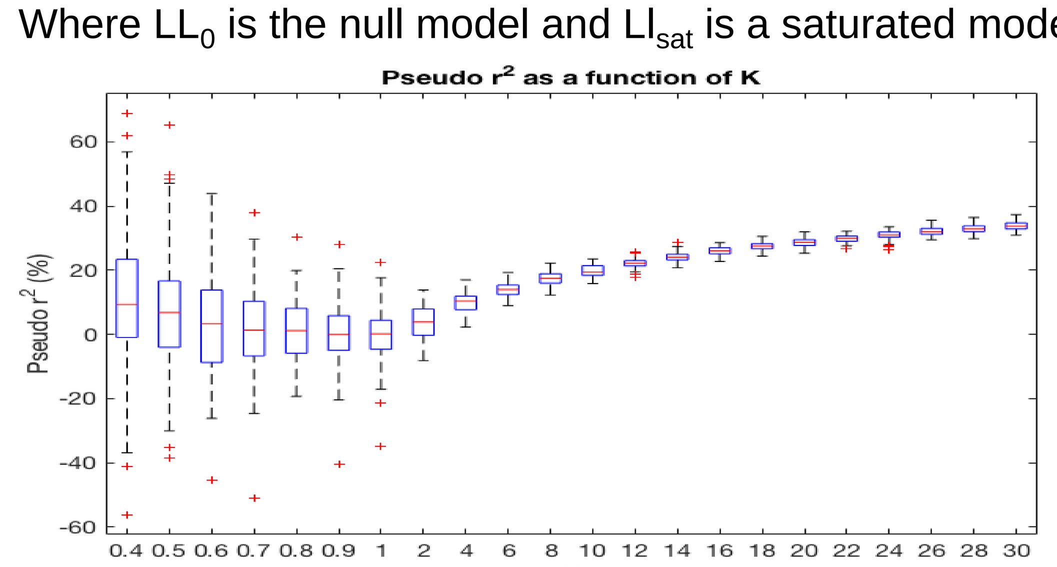

Figure 7: Relationship between the pseudo r^2 and the WRP model. Each pseudo r^2 was calculated under a test of trial data and each trial had the same number of spikes. We show that as data becomes more under dispersed (k>1), the pseudo r^2 increases to about 33%. For over dispersed data (k < 1) the pseudo r^2 increases but has a much larger spread.

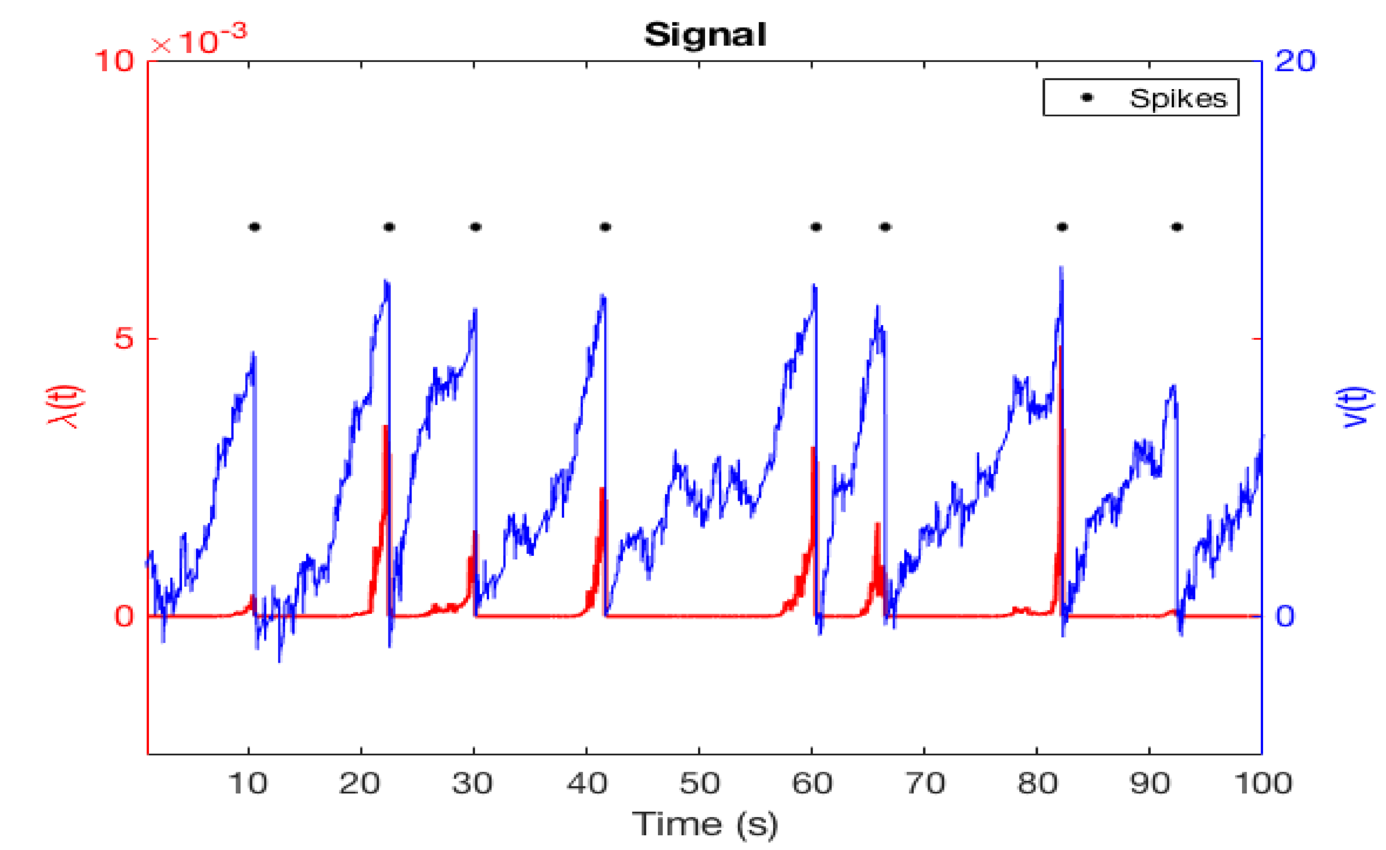

Figure 4: WRP generated data with random stimulus applied. If a spike occurs, the membrane potential is reset to 0. **Red:** Conditional Intensity function over time. **Blue:** The 'membrane potefntial' over time.

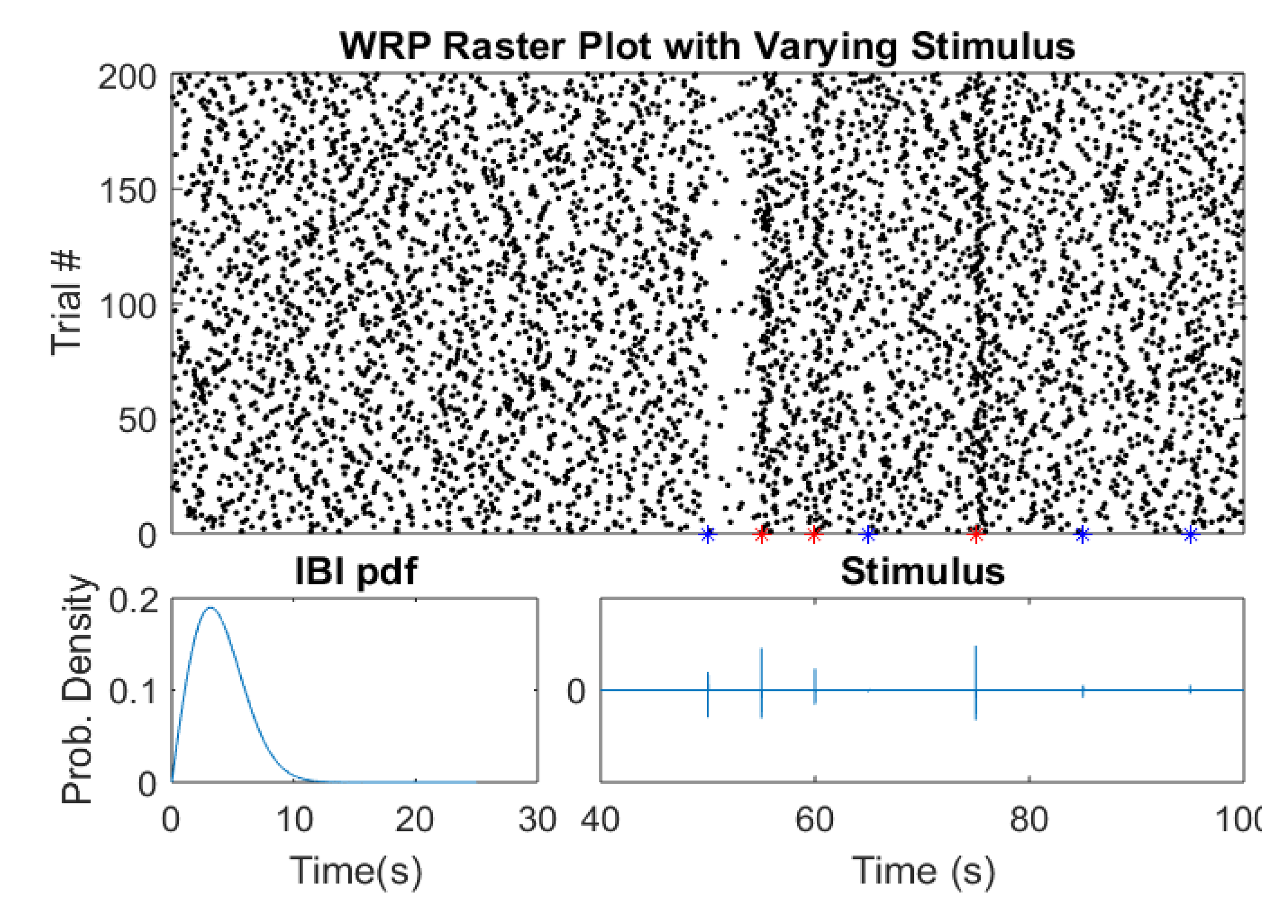

Figure 5: WRP generated data of 200 trials over 100 seconds with parameters k = 2, a = 0.0982, a mean firing rate of about 0.25Hz and a stimulus with a varying strength. **Top:** Raster plot of the data. From time 0 to 50 seconds there is no stimulus. Blue and red star represents an inhibitory/excitatory stimulus respectively. The stronger the stimulus the more apparent the effects on the spiking time. **Bottom left:** Spontaneous Inter-burst-interval probability distribution. **Bottom right:** Filtered stimulus applied to system.

Conclusion

1) Proposed a new model, the WRP model, that is a point process renewal model whose spontaneous IBI can be described by a Weibull Distribution.
2) Model uses a fairly compact parameter space and has a nonconvex negative likelihood.
3) The model can deal with both over and under dispersed data and appears to outperforms that of the homogenous Poisson model.
4) From here we aim to compare this model against other models, especially that of the GLM.
5) Future steps also include using the model to decode real data and see how well it can capture the applied stimulus.