Adaptive Solution Transfer Between Non-matching Meshes for Coupling Climate Models

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Introduction
Solution transfer plays an important role in climate modeling and many other multiphysics coupling problems. We need to transfer data across the interface between two non-matching meshes. The main focus is to transfer solutions in high-order accuracy, with robust treatment of discontinuities.

We propose an adaptive method, which supports both nodal and cell-averaged data, can achieve high-order super-convergence in smooth regions, and can detect and resolve discontinuities.

Mathematical Foundation

Weighted Least Squares (WLS) for Scattered Data Points
- Fundamental idea is to construct Taylor polynomial from scattered source nodes in a small neighborhood about a query point \( w_i \), i.e.
  \[
g(w) = \sum_{j=0}^{p} \sum_{i=0}^{n} c_{ij}(n - u_i)^j(v - v_j)^j + O((|n| - u_i)^{p+1})
\]

- To transfer data on surfaces, construct Taylor polynomial in (approximate) tangent plane.

- For stability, number of nodes in the stencil of Taylor polynomial is about 1.5 – 2 times of the number of coefficients, where each node is assigned a distance-based weight (later).

- Resulting generalized Vandermonde system is solved with truncated QR with column pivoting (QRCP) [1].

Mesh Association for Linear-Time Complexity
- To achieve optimal efficiency, we construct stencils by leveraging the mesh connectivity and searching from neighbor to neighbor, which requires constant time per node in amortized cost [2], compared to logarithmic time with tree-based data structures.

Superconvergence for Smooth Functions
- Like interpolation, WLS with degree-\( p \) polynomial basis functions delivers \( (p + 1) \) order of accuracy in general. However, it superconverges at \( (p + 2) \) order for even-degree \( p \) with symmetric stencils.

- Superconvergence can be observed for nearly symmetric stencils with a good weighting schemes, such as the Vandermonde's interpolation for even-degree and symmetric stencils.

Adaptive Solution Transfer for Nodal Data

Overall Algorithm
- Obtain an intermediate solution by applying WLS pointwise, assuming smooth functions.
- Detect discontinuous regions with our discontinuity indicator.
- Resolve discontinuous regions using essentially non-oscillatory weighting schemes.
- Recover (approximate) local conservation in discontinuous region (optional).

Step I: Detect Discontinuous Regions
- Compute discontinuity indicator at each target node based on WLS solution \( f_j \) and linear interpolation \( \tilde{f}_j \), where \( \delta_j \) is local maximum of \( |f_j| \) and \( \tilde{f}_j \) is the local maximal edge length.
- Mark target node \( i \) and its one-ring neighborhood as discontinuous candidate if \( u_i > \delta \) for some \( \delta_i \)

Step II: Detect Discontinuous Regions
- Compute discontinuity indicator at each target node based on WLS solution \( f_j \) and linear interpolation \( \tilde{f}_j \), where \( \delta_j \) is local maximum of \( |f_j| \) and \( \tilde{f}_j \) is the local maximal edge length.
- Mark target node \( i \) and its one-ring neighborhood as discontinuous candidate if \( u_i > \delta \) for some \( \delta_i \)

Step III: Resolve Discontinuities
- Recompute WLS for each target node in discontinuous region using an essentially non-oscillatory weighting scheme, similar to WLS-END [4], i.e.
  \[
v_j = \Bigg( f_j - f_j \Bigg) + \gamma_j^2 \Bigg( (u_i - u_j)^2 + (v_i - v_j)^2 \Bigg)^{p/2}
\]

where \( \gamma_j = \max(n_i, n_j, \delta) \).

Step IV: Recover Local Conservation
- Optionally we apply an iterative, monotonically-preserving step to recover (approximate) local conservation near discontinuity regions, based on a generalized weighted residuals formulation.

Extension to Cell-Averaged Data

Comparative Study

Test Meshes
- Use cubed-sphere mesh and triangle mesh for nodal solution transfer.
- Use spherical centroidal Voronoi tessellations (SCVT) for cell-averaged data.

Numerical Results

Conclusions and Future Work
- We have developed an adaptive method for data transfer based on weighted least squares for smooth and discontinuous functions. We are working on approximate local conservation for discontinuous regions, and will perform more systematic comparisons in multiphysics simulations.

References

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